NSW Education Standards Authority

2021 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number on the Question 13 Writing Booklet attached

Total marks: 100

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7–16)

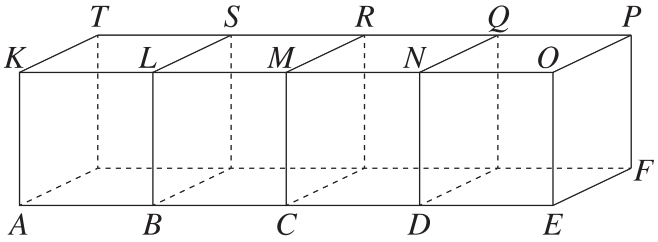
- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 Four cubes are placed in a line as shown on the diagram.



Which of the following vectors is equal to $\overrightarrow{AB} + \overrightarrow{CQ}$?

A. $\overrightarrow{AQ}$

B. $\overrightarrow{CP}$

C. $\overrightarrow{PB}$

D. $\overrightarrow{RA}$

2 Which expression is equal to $\int x^5 e^{7x}\,dx$?

A. $\frac{1}{7}x^5e^{7x} - \frac{5}{7}\int x^4e^{7x}\,dx$

B. $\frac{1}{7}x^5e^{7x} - \frac{5}{7}\int x^5e^{7x}\,dx$

C. $\frac{5}{7}x^4e^{7x} - \frac{5}{7}\int x^4e^{7x}\,dx$

D. $\frac{5}{7}x^4e^{7x} - \frac{5}{7}\int x^5e^{7x}\,dx$

3 Which of the following is a vector equation of the line joining the points $A(4, 2, 5)$ and $B(-2, 2, 1)$?

A. $\underset{\sim}{r} = \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

B. $\underset{\sim}{r} = \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$

C. $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix}$

D. $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix}$

4 Consider the statement:

‘For all integers *n*, if *n* is a multiple of 6, then *n* is a multiple of 2’.

Which of the following is the contrapositive of the statement?

A. There exists an integer *n* such that *n* is a multiple of 6 and not a multiple of 2.

B. There exists an integer *n* such that *n* is a multiple of 2 and not a multiple of 6.

C. For all integers *n*, if *n* is not a multiple of 2, then *n* is not a multiple of 6.

D. For all integers *n*, if *n* is not a multiple of 6, then *n* is not a multiple of 2.

5 Which of the following statements is FALSE?

A. $\forall\, a, b \in \mathbb{R}, \quad a < b \Rightarrow a^3 < b^3$

B. $\forall\, a, b \in \mathbb{R}, \quad a < b \Rightarrow e^{-a} > e^{-b}$

C. $\forall\, a, b \in (0, +\infty), \quad a < b \Rightarrow \ln a < \ln b$

D. $\forall\, a, b \in \mathbb{R}$, with $a, b \neq 0, \quad a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$

6 Which polynomial could have $2 + i$ as a zero, given that k is a real number?

A. $x^3 - 4x^2 + kx$

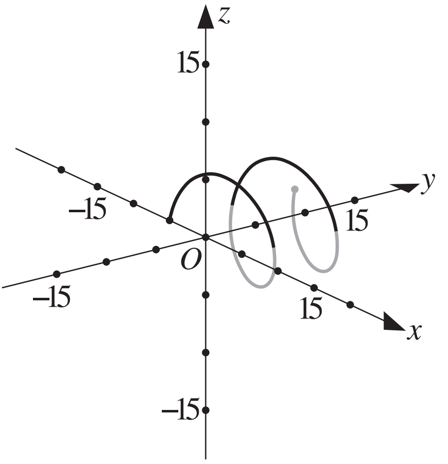
B. $x^3 - 4x^2 + kx + 5$

C. $x^3 - 5x^2 + kx$

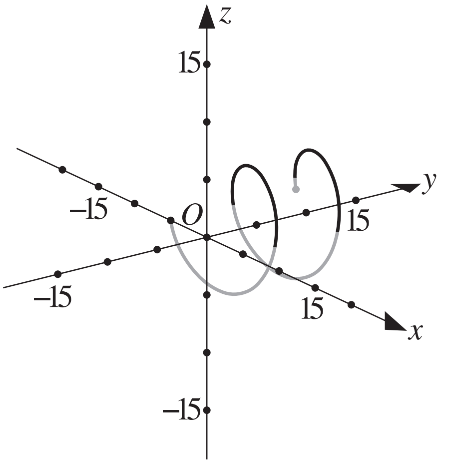
D. $x^3 - 5x^2 + kx + 5$

7 Which diagram best shows the curve described by the position vector
$\underset{\sim}{r}(t) = -5\cos(t)\underset{\sim}{i} + 5\sin(t)\underset{\sim}{j} + t\underset{\sim}{k}$ for $0 \le t \le 4\pi$?

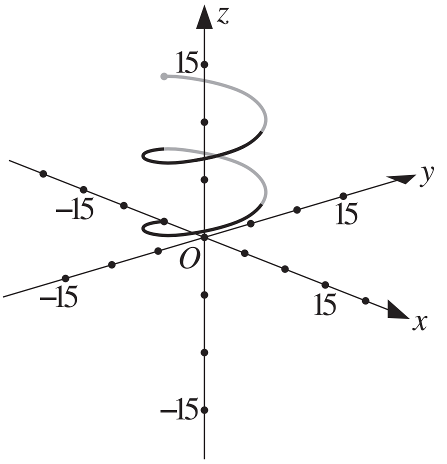
A.



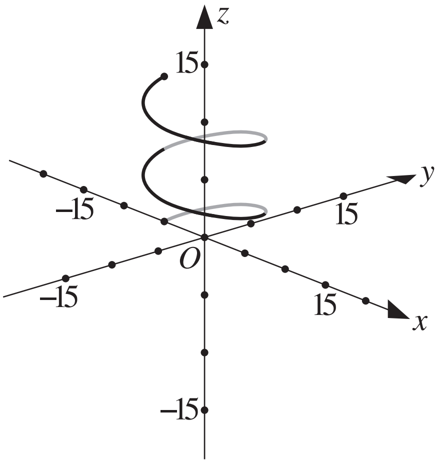
B.



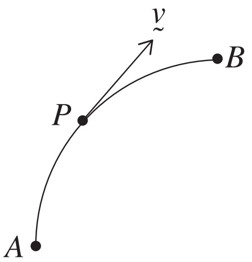
C.



D.



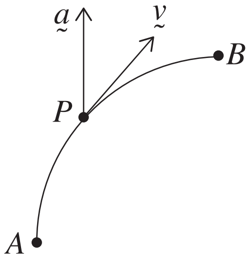
8 A particle is travelling from A on the curve joining A to B. At a particular time, the particle is at point P and has velocity $\underset{\sim}{v}$, as shown in the diagram.



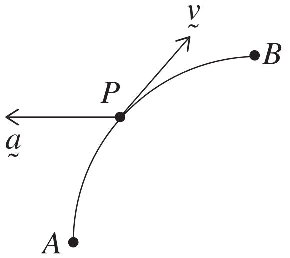
The speed of the particle is increasing.

Which of the following diagrams shows an acceleration, $\underset{\sim}{a}$, which would allow the particle to follow the curve to B?

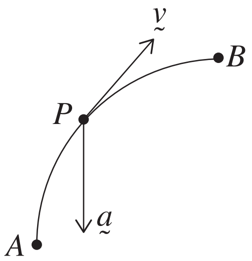
A.



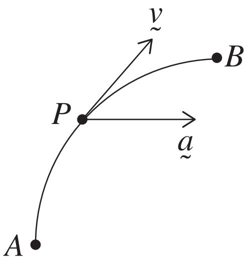
B.



C.

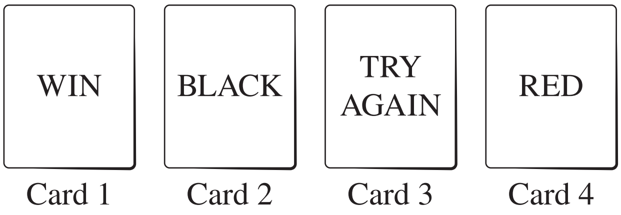


D.



9 Four cards have either RED or BLACK on one side and either WIN or TRY AGAIN on the other side.

Sam places the four cards on the table as shown below.



A statement is made: ‘If a card is RED, then it has WIN written on the other side’.

Sam wants to check if the statement is true by turning over the minimum number of cards.

Which cards should Sam turn over?

A. 1 and 4

B. 3 and 4

C. 1, 2 and 4

D. 1, 3 and 4

10 Consider the two non-zero complex numbers z and w as vectors.

Which of the following expressions is the projection of z onto w?

A. $\dfrac{\operatorname{Re}(zw)}{|w|}w$

B. $\left|\dfrac{z}{w}\right|w$

C. $\operatorname{Re}\left(\dfrac{z}{w}\right)w$

D. $\dfrac{\operatorname{Re}(z)}{|w|}w$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use the Question 11 Writing Booklet

(a) The complex numbers $z = 2e^{i\frac{\pi}{2}}$ and $w = 6e^{i\frac{\pi}{6}}$ are given. **2**

Find the value of zw, giving the answer in the form $re^{i\theta}$.

(b) Find $\sum_{n=1}^{5} (i)^n$. **2**

(c) Find the angle between the vectors $\underset{\sim}{a} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$, giving the angle in degrees correct to 1 decimal place. **3**

(d) (i) Find the two square roots of $-i$, giving the answers in the form $x + iy$, where x and y are real numbers. **2**

(ii) Hence, or otherwise, solve $z^2 + 2z + 1 + i = 0$ giving your solutions in the form $a + ib$ where a and b are real numbers. **2**

(e) The complex numbers $z = 5 + i$ and $w = 2 - 4i$ are given. **2**

Find $\frac{\overline{z}}{w}$, giving your answer in Cartesian form.

(f) Express $\frac{3x^2 - 5}{(x-2)(x^2+x+1)}$ as a sum of partial fractions over $\mathbb{R}$. **3**

Question 12 (15 marks) Use the Question 12 Writing Booklet

(a) Find $\displaystyle\int \frac{2x+3}{x^2+2x+2}\,dx$. **3**

(b) Consider Statement A.

Statement A: 'If n^2 is even, then n is even.'

(i) What is the converse of Statement A? **1**

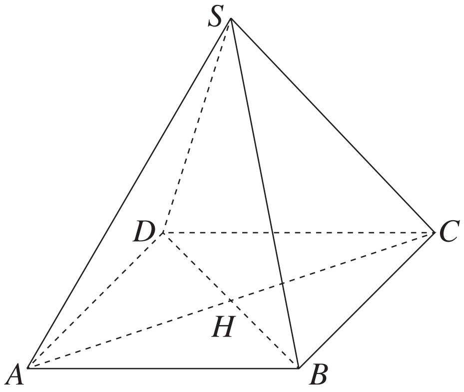
(ii) Show that the converse of Statement A is true. **1**

(c) Two lines are given by $\mathbf{r}_1 = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 4 \\ -2 \\ q \end{pmatrix} + \mu \begin{pmatrix} p \\ 3 \\ -1 \end{pmatrix}$, where p and q are real numbers. These lines intersect and are perpendicular. **3**

Find the values of p and q.

(d) Prove by mathematical induction that $\sqrt{n!} > 2^n$, for integers $n \geq 9$. **3**

(e) The diagram shows the pyramid $ABCDS$ where $ABCD$ is a square. The diagonals of the square bisect each other at H.



(i) Show that $\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD} = \underset{\sim}{0}$. **1**

Let G be the point such that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + \overrightarrow{GD} + \overrightarrow{GS} = \underset{\sim}{0}$.

(ii) Using part (i), or otherwise, show that $4\overrightarrow{GH} + \overrightarrow{GS} = \underset{\sim}{0}$. **2**

(iii) Find the value of λ such that $\overrightarrow{HG} = \lambda\overrightarrow{HS}$. **1**

Question 13 (15 marks) Use the Question 13 Writing Booklet

(a) The location of the complex number $a + ib$ is shown on the diagram on page 1 of the Question 13 Writing Booklet. **2**

On the diagram provided in the writing booklet, indicate the locations of all of the fourth roots of the complex number $a + ib$.

(b) Use an appropriate substitution to evaluate $\displaystyle\int_{\sqrt{10}}^{\sqrt{13}} x^3\sqrt{x^2 - 9}\,dx$. **3**

Question 13 continues on page 10

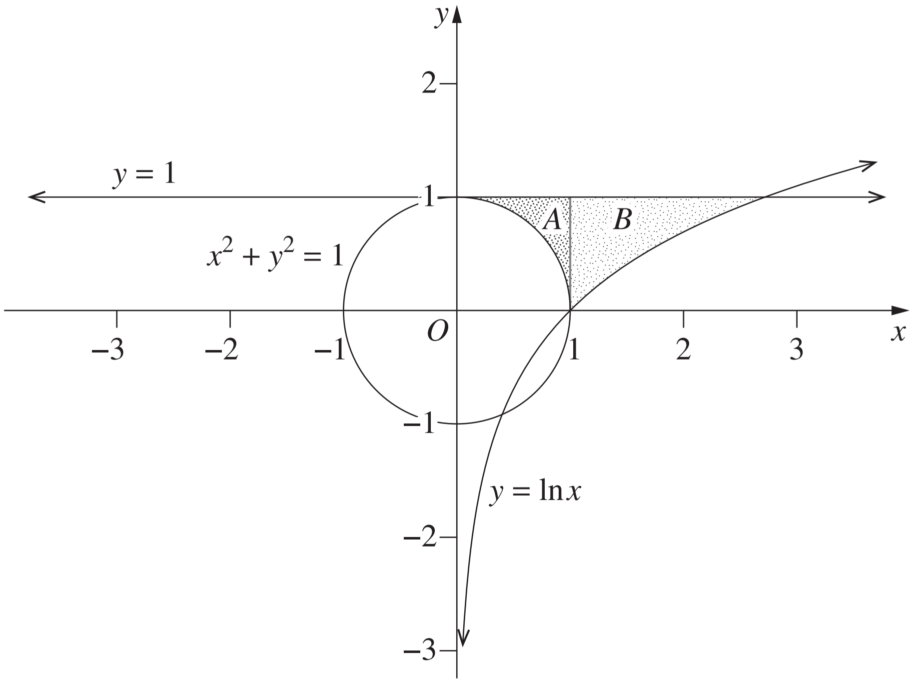
(c) (i) The integral I_n is defined by $I_n = \int_1^e (\ln x)^n \, dx$ for integers $n \geq 0$. **2**

Show that $I_n = e - nI_{n-1}$ for $n \geq 1$.

(ii) The diagram shows two regions. **4**

Region *A* is bounded by $y = 1$ and $x^2 + y^2 = 1$ between $x = 0$ and $x = 1$.

Region *B* is bounded by $y = 1$ and $y = \ln x$ between $x = 1$ and $x = e$.



The volume of the solid created when the region between the curve $y = f(x)$ and the x-axis, between $x = a$ and $x = b$, is rotated about the x-axis is given by $V = \pi \int_a^b [f(x)]^2 \, dx$.

The volume of the solid of revolution formed when region *A* is rotated about the x-axis is V_A.

The volume of the solid of revolution formed when region *B* is rotated about the x-axis is V_B.

Using part (i), or otherwise, show that the ratio $V_A : V_B$ is $1:3$.

Question 13 continues on page 11

(d) An object is moving in simple harmonic motion along the x-axis. The acceleration of the object is given by $\ddot{x} = -4(x - 3)$ where x is its displacement from the origin, measured in metres, after t seconds.

Initially, the object is 5.5 metres to the right of the origin and moving towards the origin. The object has a speed of 8 m s^{-1} as it passes through the origin.

(i) Between which two values of x is the particle oscillating? **2**

(ii) Find the first value of t for which $x = 0$, giving the answer correct to 2 decimal places. **2**

End of Question 13

Please turn over

Question 14 (14 marks) Use the Question 14 Writing Booklet

(a) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{3+5\cos x}\,dx$. **4**

(b) An object of mass 5 kg is on a slope that is inclined at an angle of 60° to the horizontal. The acceleration due to gravity is g m s^{-2} and the velocity of the object down the slope is v m s^{-1}.

As well as the force due to gravity, the object is acted on by two forces, one of magnitude $2v$ newtons and one of magnitude $2v^2$ newtons, both acting up the slope.

(i) Show that the resultant force down the slope is **2**

$\dfrac{5\sqrt{3}}{2}g - 2v - 2v^2$ newtons.

(ii) There is one value of v such that the object will slide down the slope at a constant speed. **2**

Find this value of v in m s^{-1}, correct to 1 decimal place, given that $g = 10$.

(c) (i) Using de Moivre's theorem and the binomial expansion of $(\cos\theta + i\sin\theta)^5$, or otherwise, show that **3**

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

(ii) By using part (i), or otherwise, show that $\operatorname{Re}\left(e^{\frac{i\pi}{10}}\right) = \sqrt{\dfrac{5+\sqrt{5}}{8}}$. **3**

Question 15 (15 marks) Use the Question 15 Writing Booklet

(a) For all non-negative real numbers x and y, $\sqrt{xy} \le \frac{x+y}{2}$. (Do NOT prove this.)

(i) Using this fact, show that for all non-negative real numbers a, b and c, **2**

$$\sqrt{abc} \le \frac{a^2 + b^2 + 2c}{4}.$$

(ii) Using part (i), or otherwise, show that for all non-negative real numbers a, b and c, **2**

$$\sqrt{abc} \le \frac{a^2 + b^2 + c^2 + a + b + c}{6}.$$

(b) For integers $n \ge 1$, the triangular numbers t_n are defined by $t_n = \frac{n(n+1)}{2}$, giving $t_1 = 1$, $t_2 = 3$, $t_3 = 6$, $t_4 = 10$ and so on.

For integers $n \ge 1$, the hexagonal numbers h_n are defined by $h_n = 2n^2 - n$, giving $h_1 = 1$, $h_2 = 6$, $h_3 = 15$, $h_4 = 28$ and so on.

(i) Show that the triangular numbers t_1, t_3, t_5, and so on, are also hexagonal numbers. **2**

(ii) Show that the triangular numbers t_2, t_4, t_6, and so on, are not hexagonal numbers. **1**

Question 15 continues on page 14

Question 15 (continued)

(c) An object of mass 1 kg is projected vertically upwards with an initial velocity of u m/s. It experiences air resistance of magnitude kv^2 newtons where v is the velocity of the object, in m/s, and k is a positive constant. The height of the object above its starting point is x metres. The time since projection is t seconds and acceleration due to gravity is g m/s^2.

(i) Show that the time for the object to reach its maximum height is $\frac{1}{\sqrt{gk}}\arctan\left(u\sqrt{\frac{k}{g}}\right)$ seconds. **3**

(ii) Find an expression for the maximum height reached by the object, in terms of k, g and u. **3**

(d) Prove that $2^n + 3^n \neq 5^n$ for all integers $n \geq 2$. **2**

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet

(a) (i) The point $P(x, y, z)$ lies on the sphere of radius 1 centred at the origin O. **2**

Using the position vector of P, $\overrightarrow{OP} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$, and the triangle inequality, or otherwise, show that $|x| + |y| + |z| \geq 1$.

(ii) Given the vectors $\underset{\sim}{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, show that **3**

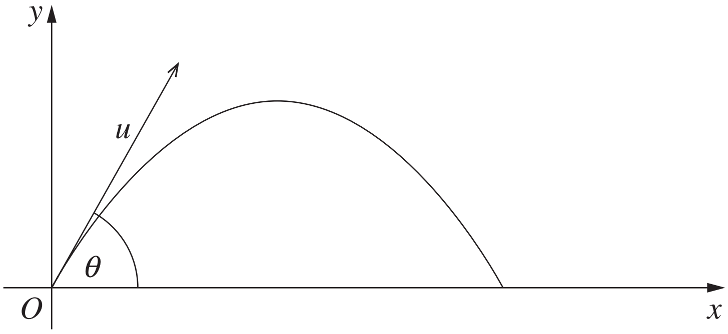
$$|a_1b_1 + a_2b_2 + a_3b_3| \leq \sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}\,.$$

(iii) As in part (i), the point $P(x, y, z)$ lies on the sphere of radius 1 centred at the origin O. **2**

Using part (ii), or otherwise, show that $|x| + |y| + |z| \leq \sqrt{3}$.

Question 16 continues on page 16

(b) A particle which is projected from the origin with initial speed u m s^{-1} at an angle of θ to the positive x-axis lands on the x-axis, as shown in the diagram. The particle is subject to an acceleration due to gravity of g m s^{-2}. **5**



The position vector of the particle, $\underset{\sim}{r}(t)$, where t is the time in seconds after the particle is projected, is given by

$$\underset{\sim}{r}(t) = \begin{pmatrix} ut\cos\theta \\ -\frac{gt^2}{2} + ut\sin\theta \end{pmatrix}. \qquad \text{(Do NOT prove this.)}$$

For some value(s) of θ there will be two times during the time of flight when the particle's position vector is perpendicular to its velocity vector.

Find the value(s) of θ for which this occurs, justifying that both times occur during the time of flight.

(c) Sketch the region of the complex plane defined by $\text{Re}(z) \geq \text{Arg}(z)$ where $\text{Arg}(z)$ is the principal argument of z. **3**

End of paper

NSW GOVERNMENT

NSW Education Standards Authority

Centre Number

Student Number

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

Writing Booklet

Question 13

Instructions

- Use this Writing Booklet to answer Question 13.
- Write the number of this booklet and the total number of booklets that you have used for this question (eg: 1 of 3).
- Write your Centre Number and Student Number at the top of this page.
- Write using black pen.
- You may ask for an extra writing booklet if you need more space.
- If you have not attempted the question(s), you must still hand in the writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover.
- You may NOT take any writing booklets, used or unused, from the examination room.

13

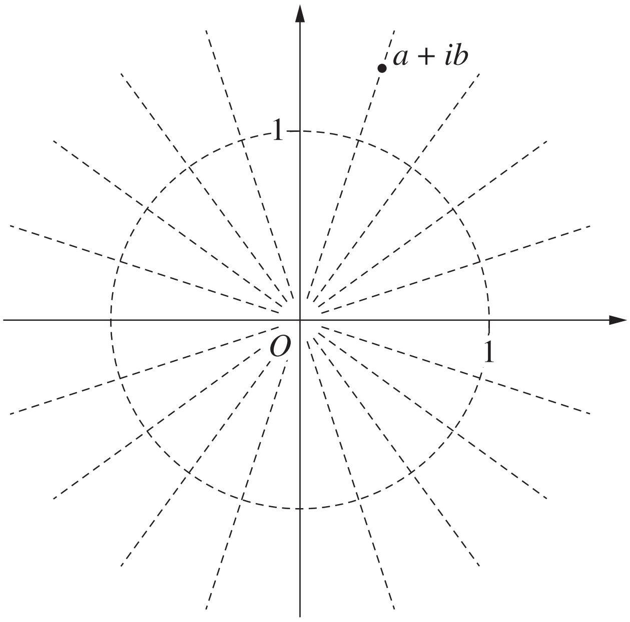
of

this booklet

number of booklets for this question

Start here for
Question Number: **13**

(a) Indicate the locations of all of the fourth roots of the complex number $a + ib$.



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☐ ⇦ Tick this box if you have continued this answer in another writing booklet.

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

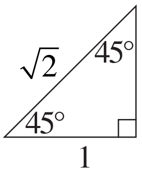
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

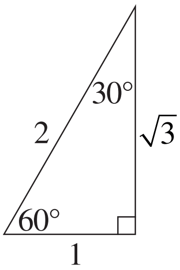


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

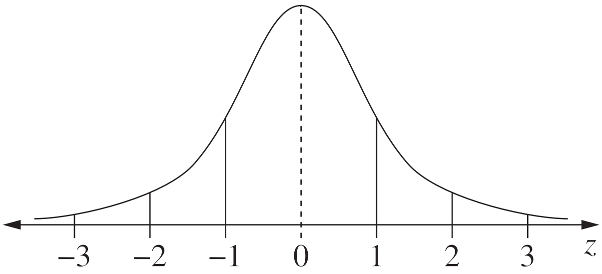
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	**Derivative**
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2\,,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^{2}(x - c)$$